PHYCC-1 I (Unit I) Plasma Physics

[Magnetic Mirrors, Pinch Effect and Adiabatic Expansion] 20-4-2020

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Plasma Confinement and Magnetic Mirrors :—

So far, in our discussion of Motion of Particle under Simultaneous Electric and Magnetic field (Uniform 'OR' Non-uniform) we have seen :-

(i) If the charge particle moves (E=0) in the direction $\perp_r$ to $\vec{B}$, then Larmor frequency $(\omega_B) = \frac{eH}{B}$ and $r = \frac{mv}{eB}$

(ii) If velocity of the particle in (E=0) is not perpendicular to magnetic field, then v is replaced by $v_\perp$ (component of velocity parallel to magnetic field) then

$$r = \frac{mv_\perp}{eB} \quad \text{and } \omega_B = \frac{eH}{m}$$
= Gyrofrequency.

Resulting Path is Helical with guiding Centre lying on the line of force.

(iii) Motion of charge particle ($E \neq 0$ along E_x) and B along y-axis simultaneously, then

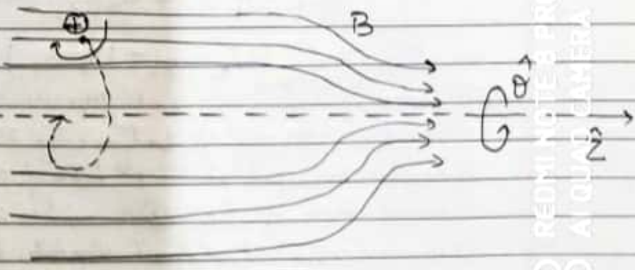
$$V_x = V_\perp e^{j\omega_H t} \quad v_y = v_\perp e^{j\omega t} - \frac{E}{B}$$

The Larmor frequency is the same as in earlier cases but there is superimposed drift v_d of the guiding Centre in the negative y-direction.

(iv) In case of Inhomogeneous magnetic field, the Larmour ~~f~~ radius $r = \frac{mv_\perp}{eB}$ gets changed due to gradient in magnetic field.

The oppositely charged particles (electrons and Ions) will drift in the opposite directions and this will result in a charge - separation. ~~of~~

A plasma System, Consisting of electrons and ions will gyrate about the magnetic lines of force. Let us Consider a magnetic field along Z-axis, whose magnitude varies in Z-direction. Let the field be axisymmetric with $B_\theta = 0$ and $\frac{\partial}{\partial \theta} = 0$. Since line of forces converge and diverge, there is necessarily a component B_r



From $\vec{\nabla}\cdot\vec{B} = 0$

we get $\frac{1}{r}\frac{\partial}{\partial r}(r B_r) + \frac{\partial B_z}{\partial z} = 0$

If $\frac{\partial B_z}{\partial z} = 0$ is given at $r = 0$ and doesnot vary much with r, we have

$$r B_r = -\int_0^r r \frac{\partial B_z}{\partial z}\, dr \simeq -\frac{1}{2} r^2 \left[\frac{\partial B_z}{\partial z}\right]_{r=0}$$

$$\Rightarrow \boxed{B_r = -\frac{1}{2} r \left[\frac{\partial B_z}{\partial z}\right]_{r=0}}$$

This variation of $|B|$ with r Causes a grad-B drift of guiding Centres about axis of Symmetry, but there is no radial grad-B drift, because $\frac{\partial B}{\partial \theta} = 0$.

The Component of the Lorentz force are,

$$F_r = q(v_\theta B_z - v_z B_\theta) \quad \text{--- (i)}$$

$$F_\theta = q(-v_r B_z + v_z B_r) \quad \text{--- (ii)}$$

$$F_z = q(v_z B_\theta - v_\theta B_r) \quad \text{--- (iii)}$$

If $B_\theta = 0$, First term eqn (i) and eqn(ii) give rise to Larmor gyration. The 2nd term of eqn (i) vanishes with 1st term of eqn. (iii). Second term of Eqn. (2) vanishes on the axis and when it doesnot vanish, the azimuthal force Cause a drift in radial direction. This drift only makes the guiding Centers to follow the lines of force.

For clear understanding of this mathematics, let us consider the plasma to be produced in a Cylinder. The loss of ions due to Collision be Compensated by Collision with the wall of the Container (end Walls). The magnetic field be applied across the Cylinder by wrapping Winding Coils, through which current is passed to generate $\vec{B}$.

we Know $\vec{B} = \frac{2I}{r}$ directed along the axis of the torus and the field is inhomogeneous in the direction $\perp$ to field direction.

Lines of force are curved and hence $\vec{B}$ has two types of inhomogeneity — (i) variation of $\vec{B}$ along radial direction (ii) variation due to curvature lines of force. These two effects Combined together to produce drift of electrons and ions.

$F = \frac{m v_{\parallel}^2}{e r}$, $v_{\parallel}$ = velocity along line of force.

$B = \frac{2I}{r} \Rightarrow \frac{dB}{dr} = -\frac{2I}{r^2} = -\frac{B}{r}$ and $r_\perp = \frac{m v_\perp}{eB}$

so, drift due to radial inhomogeneity is given by

$$= \frac{v_\perp r_\perp}{2B}\frac{dB}{dr} = \frac{m v_\perp^2}{2eB^2}\frac{B}{r} = \frac{W_\perp}{eBR}$$

$$W_\perp = \frac{1}{2} m v_\perp^2$$

[where R = radius of the Tube]

$\therefore$ The total drift $= \dfrac{W_\perp}{eBr} + \dfrac{2W_{\parallel}}{eBR} = \dfrac{(W_\perp + 2W_\parallel)}{eBR}$

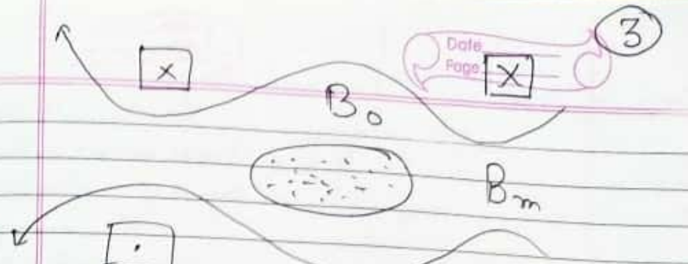
For Maxwellian distribution

$$W_\perp = KT_e \quad \text{and} \quad W_\parallel = \frac{kT_e}{2}$$

$$\therefore \text{Total drift} = \frac{2kT_e}{eBR}$$

These drift in opposite directions for electrons and ions creat an electric field, in conjunction with the magnetic field will cause the plasma system, as a whole, to drift in the direction perpendicular to both the fields. Since the drift is inversely proportional to both to both $\vec{B}$ and major radius of torus, the drift can be reduced by increasing $\vec{B}$ and R.

As a particle moves from a weak field region to a strong field Region, in the Course of its thermal motion, with increase in B, the particle accquires high $v_\perp$. In ~~At~~ such situation of increasing $v_\perp$, the Corrosponding $v_\parallel$ must necessarily decrease to maintain Constant Energy of the System. If in the throat of the mirror B is high, $v_\parallel \to 0$ and the particle gets "reflected" back in to the weak field region, due to $F_\parallel$.



[A Plasma trapped between Magnetic Mirrors]

$\therefore$ The non-uniform field of a simple pair of coils forms two magnetic mirrors between which a plasma can be trapped, which can't be perfect as ~~for~~ particles with $v_\perp = 0$, no magnetic moment or Force can be experienced. Such particles having $v_\perp \ll v_\parallel$ at the mid plane ($B = B_0$) will also escape if Max B (B_m) is not sufficiently large.

Particles with $v_\perp = v_{\perp 0}$ and $v_\parallel = v_{\parallel 0}$ at the midplane will have $v_\perp = v_\perp'$ and $v_\parallel = 0$ at its turning point (when $B = B'$), then

$$\frac{1}{2} m v_{\perp 0}^2 / B_0 = \frac{1}{2} m v_\perp'^2 / B'$$

and from conservation of energy, $v_\perp'^2 = v_{\perp 0}^2 + v_{\parallel 0}^2 = v_0^2$

$$\therefore \frac{B_0}{B'} = \frac{v_{\perp 0}^2}{v_\perp'^2} = \frac{v_{\perp 0}^2}{v_0^2} = \sin^2\theta$$

where θ = Pitch angle of the orbit in the weak-field reg. n.

This means that the energy density associated with an EMW in a stationary homogeneous non-conducting medium propagates with same speed with which the field vectors move.

If Plane EMW is Considered in Anisotropic (in which EM field properties depend on direction) Non-Conducting media, the energy does not propagate in the direction of wave propagation. If $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are direction Cosines of unit vector $\hat{n}$, then

$$\hat{n} = \hat{i}\cos\alpha + \hat{j}\cos\beta + \hat{k}\cos\gamma$$

and we get

$$\boxed{\frac{\cos^2\alpha}{v_x^2 - v^2} + \frac{\cos^2\beta}{v_y^2 - v^2} + \frac{\cos^2\gamma}{v_z^2 - v^2} = 0} \quad -(XV)$$

Fresnel's Law for Phase Velocity

If the anisotropic media have two directions for which only one phase velocity is soln. of eqn. (XV), these directions are Called OPTIC AXES and the medium is called BIAXIAL. Some of the medium are UNIAXIAL also, in which there is only one optic axis.

If EMW propagates in a Conducting media, no electric field can exist in such media in absence of applied current density. The Solution of Maxwell's equation yields that field amplitude (E and H) are spatially attenuated and it is measured in terms of β, known as absorption Co-efficient.

The wave speed in such media is given by

$$v = \frac{\omega}{\alpha} = \frac{1}{\sqrt{\mu\epsilon}} \cdot \left\{ \frac{\sqrt{\left\{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2\right\}} + 1}{2} \right\}^{-1/2}$$

when propagation vector $K = \alpha + i\beta$

The waves get damped exponentially (i.e. get attenuated at a distance), which is measured by β. The term $(1/\beta)$ measures the depth at which EMW entering a Conductor is damped to $\frac{1}{e} = 0.36$ of its initial amplitude at the surface. This depth is known as Skin depth or the penetration depth and is given by δ

$$\delta = \frac{1}{\beta} = \frac{1}{\omega\sqrt{\mu\epsilon}} \left[\frac{\sqrt{\left\{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2\right\}} - 1}{2} \right]^{-1/2}$$

$$\approx \sqrt{\frac{2}{\mu\sigma\omega}} \quad \left(\text{for Good Conductor } \frac{\sigma}{\omega\epsilon} << 1\right)$$

δ decreases with increasing frequency. Skin depth measures the depth to which an EMW Can penetrate in a Conducting media. Due to this thickness of Conducting Sheets, used for EM shielding, has to more than Skin depth dimension.