# The Longitudinal Adiabatic Invariant

**Course: MPHYEC-01I Plasma Physics**
**(M.Sc. IV Sem)**


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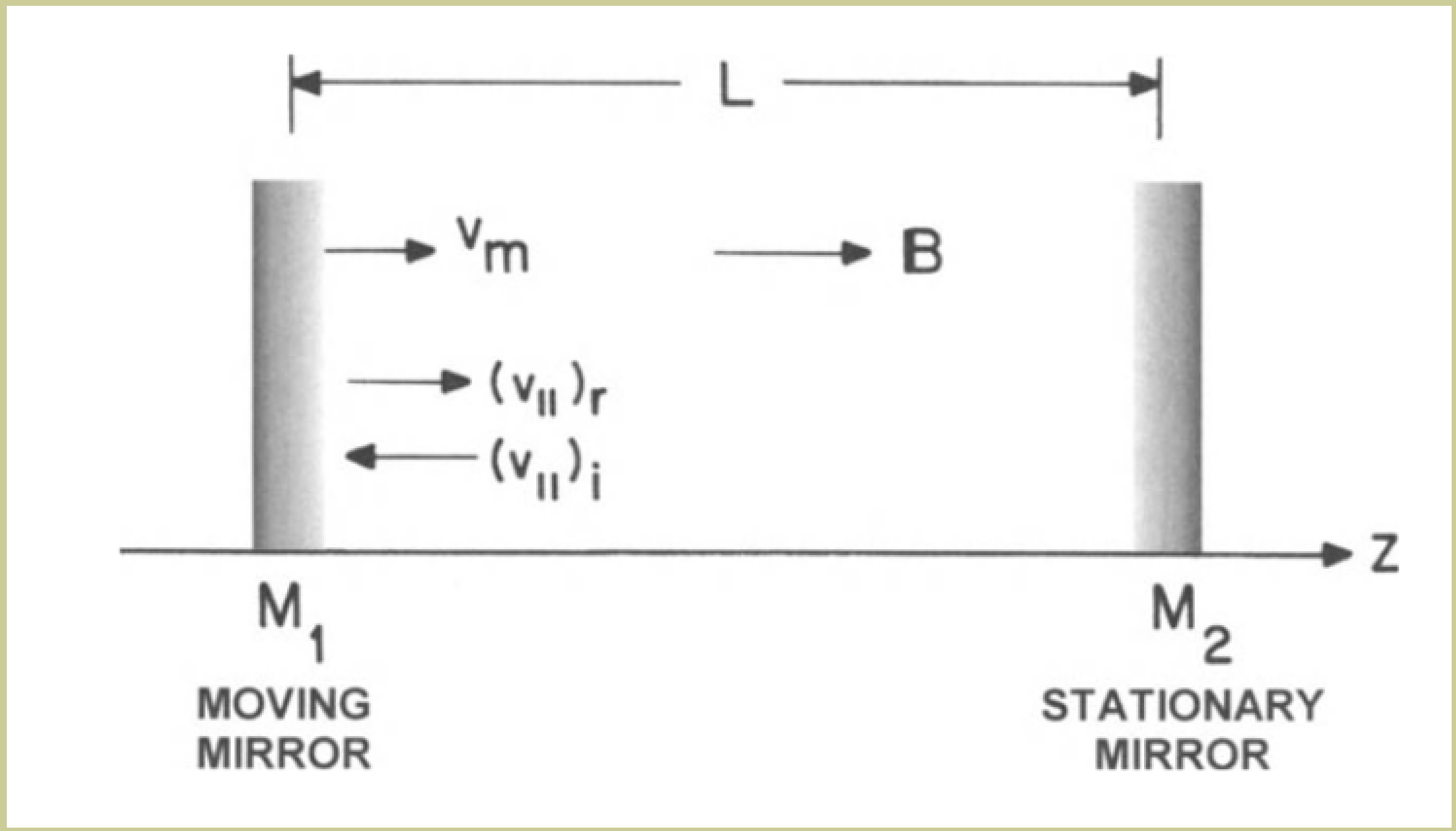
Lecture 9: Unit-I

# The Longitudinal Adiabatic Invariant

- Let's us consider that a particle is trapped between two magnetic mirrors and bouncing between them.




*Schematic representation of a system of two coaxial magnetic mirrors approaching each other. The figure is taken from the "Fundaments of the Plasma Physics" by Bittencourt.*

- **We assume that the distance between mirrors change slowly in campare to the bouncing period of the particle.**

- **With the bouncing motio of the particle between the two magnetic mirrors, there is associated an adiabatic invariant called the longitudinal adiabatic invariant, defined by the integral**

$$J = \oint \mathbf{v} \cdot d\mathbf{l} = \oint v_{\parallel} \, dl$$

taken over one period of oscillation of the particle back and forth between the mirror points.

- **Proof:** For a hand-waiving proof of the invariance of J, we consider is approximately constant along z, except near the points $M_1$ and $M_2$ (where the field strength enhances to form two mirrors).

Assume the speed at which the mirror approach to other mirror is:

$$v_m = -\frac{dL}{dt}$$

Now for slowly moving mirror $v_m \ll v_{\parallel}$ (the parallel component can be considered uniform due to uniform nature of B inside the mirrors). Then

$$J = \int_0^{2L} v_{\parallel} \, dl = 2 v_{\parallel} \, L$$

The rate of change of J is:

$$\frac{dJ}{dt} = 2v_{\parallel}\ \frac{dL}{dt} + 2L\frac{dv_{\parallel}}{dt} = -2v_{\parallel}\ v_m + 2L\frac{dv_{\parallel}}{dt}$$

$$\frac{dv_{\parallel}}{dt} = \frac{\Delta v_{\parallel}}{\Delta t} = \frac{\Delta v_{\parallel}}{(2L/v_{\parallel}\ )}$$

The change in the particle speed, in one reflection,

$$\Delta v_{\parallel}\ = (v_{\parallel})_r - (v_{\parallel})_i = 2v_m$$

Therefore,

$$\frac{dv_{\parallel}}{dt} = \frac{2v_m}{(2L/v_{\parallel})} = \frac{v_m v_{\parallel}}{L}$$

$$\frac{dJ}{dt} = \frac{d}{dt}(2v_{\parallel}\ L) = 0$$

The parallel kinetic energy of a charged particle trapped between the two mirrors is

$$W_{\parallel} = \frac{1}{2} m v_{\parallel}^{2} = \frac{m J^{2}}{8 L^{2}}$$

where we have considered $J = 2 v_{\parallel} L$. The energy increases with a decrease in L.

- ***Importantly, Fermi argued that this concept is central to the acceleration of charged particles in order to explain the origin of high-energy cosmic rays.***

- ***Fermi proposed that two stellar clouds moving towards each other, and having a magnetic field greater than in the space between them, may serve as magnetic mirrors and trap the cosmic charged particles which can be accelerated.***

***Thanks for the attention!***