The Concept of Debye Shielding

Course: MPHYEC-01I Plasma Physics
(M.Sc. IV Sem)

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Lecture 2: Unit-I

An Introduction to Debye Shielding

- Debye shielding occurs because electric fields associated with charged particles do work to get rid of themselves.

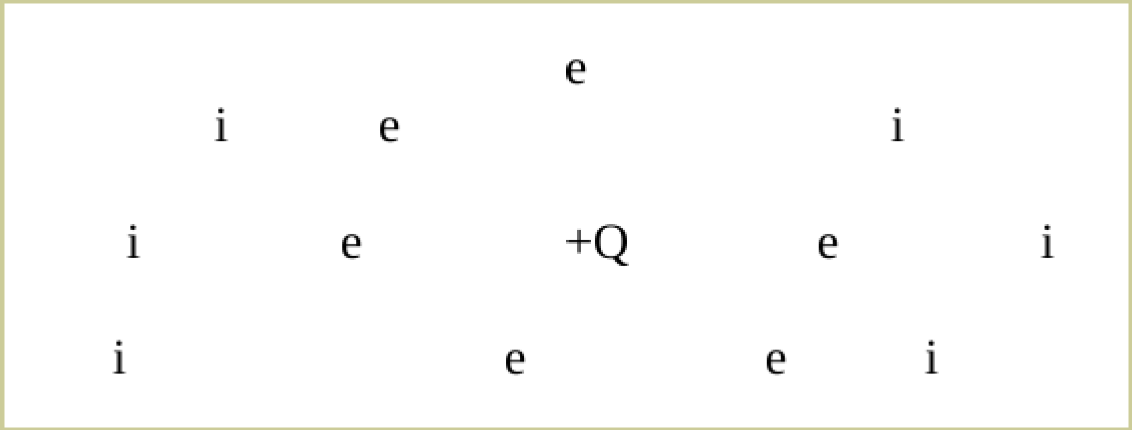
- As a result, plasmas generally do not contain strong electric fields in their rest frames.

- The Debye shielding is an important concept which quantifies the nature of plasma to respond any perturbation.

- The electrostatic field generated by the charge will be exponentially attenuated (or shielded) by a Debye length of λ_D.

Derivation of Debye Length

- To fix ideas, a test charged particle +Q is smoothly placed in the plasma which is in equilibrium state.



- Electrons get attracted and ions are repelled by the electrostatic potential ($\Phi(\mathbf{r})$) associated with the test particle.

- This leads to a slight difference in the densities of the electrons (n_e) and ions (n_i) in the proximity of the test particle, i.e. a perturbation in quasi-neutrality.

- We will derive an expression for the potential by setting-up a spherical coordinate system such that the origin concides with the location of the test particle.

- At large distances from the origin, the potential will be sufficiently weak and the quasi-neutrality ($n_i \approx n_e = n_0$) is maintained.

Derivation (Contd.)

- The plasma being in equilibrium state, the number densities of electrons and ions, will be given by the Maxwellian distributations

$$n_e(\mathbf{r}) = n_o \exp(e\Phi(\mathbf{r})/kT)$$
$$n_i(\mathbf{r}) = n_o \exp(-e\Phi(\mathbf{r})/kT)$$

  where we have considered the plasma to be at a temperature T.

- The total charged density (ρ) is given as:

$$\rho(\mathbf{r}) = -en_e + en_i + Q\delta(\mathbf{r})$$
$$= -e\, n_o (\exp(e\Phi(\mathbf{r})/kT) - \exp(-e\Phi(\mathbf{r})/kT)) + Q\delta(\mathbf{r})$$

- **Assumption**: The electrostatic energy (qΦ) is much smaller than the thermal energy (kT), i.e. $|e\Phi(r)| \ll kT$. Then

$$\rho(\mathbf{r}) = -2\, n_o\, e^2\Phi(\mathbf{r})/kT + Q\delta(\mathbf{r})$$

- From the Maxwell's first equation, electrostatic potential satifies the following Poisson's equation: $\nabla^2\Phi(\mathbf{r}) = -\rho(\mathbf{r})/\epsilon_0$

$$\Rightarrow \nabla^2\Phi(\mathbf{r}) = 2\, n_o\, e^2\Phi(\mathbf{r})/kT\varepsilon_0 + Q\delta(\mathbf{r})/\varepsilon_0$$

Derivation (Contd.)

- The Poisson's equation can be casted as:

$$\nabla^2\Phi(\mathbf{r})-2\,\Phi(\mathbf{r})/\lambda_D^2=Q\delta(\mathbf{r})/\varepsilon_0$$

  where $\lambda_D=(\varepsilon_0 kT/n_0 e^2)^{1/2}$ and denotes the **Debye length**.

- To solve this equation, the spherical symmetry of the system is exploited, as potential $\Phi_c(r)$ of an isolated point charge Q ($\Phi_c(r)=Q/4\pi\varepsilon_o r$) is spherically symmetric. Then we have

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2\frac{d\Phi(r)}{dr}\right]-\frac{2}{\lambda_D^2}\Phi(r)=0$$

- Suitable boundary conditions (BCs):

$$\Phi(r)\rightarrow\Phi_c(r) \text{ when } r\rightarrow 0$$
$$\Phi(r)\rightarrow 0 \text{ when } r\rightarrow \text{infinity}$$

- To simply the problem, we define $\Phi(r)=\Phi_c(r)F(r)=QF(r)/4\pi\varepsilon_o r$ where F(r) then satifies the BCs:

$$F(r)\rightarrow 1 \text{ when } r\rightarrow 0$$
$$F(r)\rightarrow 0 \text{ when } r\rightarrow \text{infinity}$$

- The equation in terms of F(r)

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2\frac{dF(r)}{dr}\right]-\frac{2}{\lambda_D^2}\frac{F(r)}{r}=0 \;\Rightarrow\; \frac{d^2F(r)}{dr^2}-\frac{2}{\lambda_D^2}F(r)=0$$

Derivation (Contd.)

- The general solution of the above equation is

$$F(r) = Ae^{\frac{\sqrt{2}r}{\lambda_D}} + Be^{\frac{-\sqrt{2}r}{\lambda_D}}$$

- The second boundary condition dictacts A=0 and the first boundary condition implies B=1. As a result,

$$F(r) = e^{\frac{-\sqrt{2}r}{\lambda_D}}$$

- The potential $\Phi(r) = \frac{Q}{4\pi\varepsilon_0 r} e^{\frac{-\sqrt{2}r}{\lambda_D}}$

- The potential is approximately effective to the distance $r \approx \lambda_D$ and then decays abruptly.

- A sphere of radius λ_D around the test charge is known as the Debye sphere. Potential is effective inside the sphere and outside the sphere the potential gets screened by the space charges around the test charge.

Plasma Parameter

- For a macroscopically neutral plasma, the characteristic length scale L should be much greater than λ_D.
- For a statistically meaningful treatment of plasma, the number of particles in Debye sphere should be large, i.e. $n_e \lambda_D^3 >> 1$.
- A plasma parameter g is defined as $g = 1/n_e\lambda_D^3$. Clearly, g should satisfy the condition $g << 1$.
- For Earth's ionosphere $n_e \approx 10^{12}\ m^{-3}$, $T \approx 10^3$ K then $\lambda_D \approx 10^{-3}$ m. For this case, $g = 1/10^3$ which is much less than one. So, ionospheric gas can be treated as plasma.
- **Assignment:** Calculate Debye length and plasma parameter for the solar coronal plasma, Laboratory plasma.

Thanks!