Curvature and Gradient Drift Motions

Course: MPHYEC-01I Plasma Physics
(M.Sc. IV Sem)

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Lecture 6: Unit-I

Curvature Drift

- **We consider curved field lines (without any gradient) and focus on a field line with radius of curvature R_c.**

- **Set-up a local cylinderical coordinate system such that the curved field lines are parallel to $\boldsymbol{\theta}$ direction.**

- **In the system, the components of velocity and acceleration are given as**

$$\boldsymbol{v} = \dot{r}\,\boldsymbol{e}_r + r\dot{\theta}\,\boldsymbol{e}_\theta + \dot{z}\,\boldsymbol{k}$$

$$\boldsymbol{a} = (\ddot{r} - r\dot{\theta}^2)\,\boldsymbol{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\boldsymbol{e}_\theta + \ddot{z}\boldsymbol{k}$$

- **Since field lines are assumed to be parallel to $\boldsymbol{\theta}$ direction, therefore**

$$\mathbf{B} = B_\theta e_\theta$$

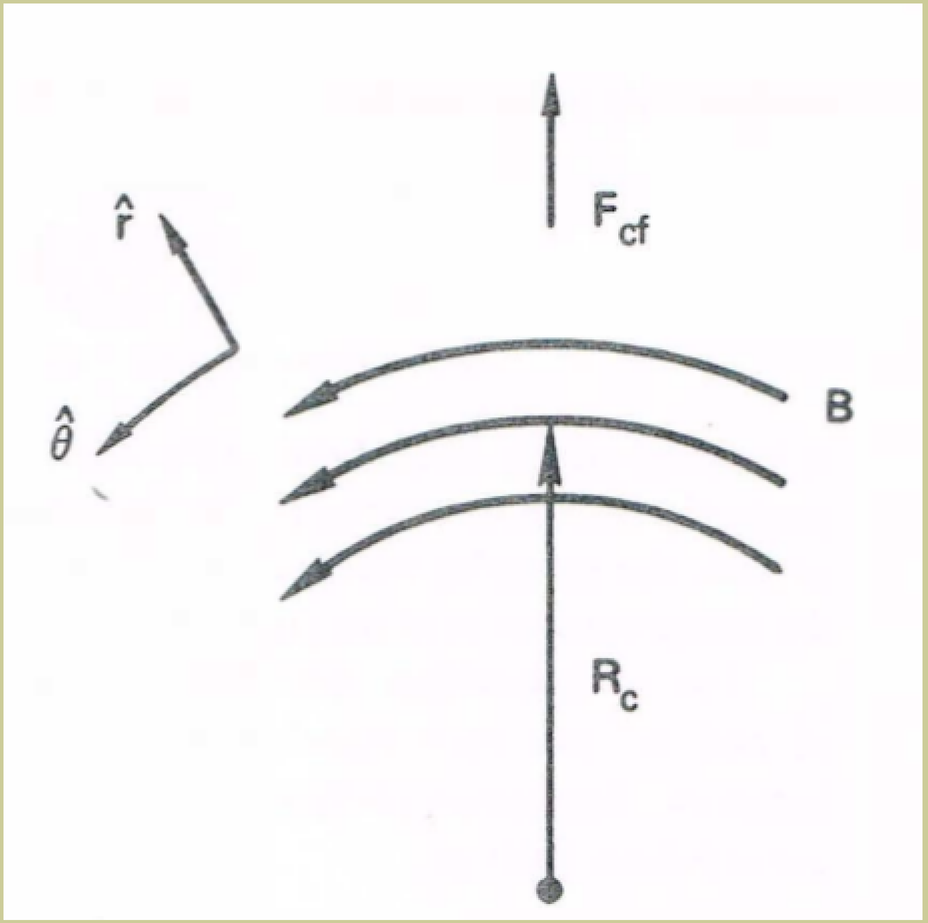

Figure is from “Introduction to Plasma Physics and Controlled Fusion” by F. F. Chen.

Equation of motion:

$$m\frac{dv_r}{dt}=-qv_zB_\theta \qquad m\frac{dv_z}{dt}=qv_rB_\theta$$

$$m\frac{d^2r}{dt^2}=-qv_zB_\theta+mr\left(\frac{d\theta}{dt}\right)^2 \rightarrow m\frac{d^2r}{dt^2}=-qv_zB_\theta+F_{cf}$$

For the field line with curvature radius R_c , $r=R_c$ and we also know that

$$v_\theta=r\frac{d\theta}{dt} = v_{||}$$

As a result,

$$F_{cf}= (m\, v_{||}^2 / R_c)$$

Curvature drift

$$\mathbf{v}_R=\frac{1}{q}\frac{\mathbf{F}_{cf}\times\mathbf{B}}{B^2}=\frac{mv_{\|}^2}{qB^2}\frac{\mathbf{R}_c\times\mathbf{B}}{R_c^2}$$

Gradient Drift

To understand the gradient drift, we consider the magnetic field B=B(y) **e**$_z$.

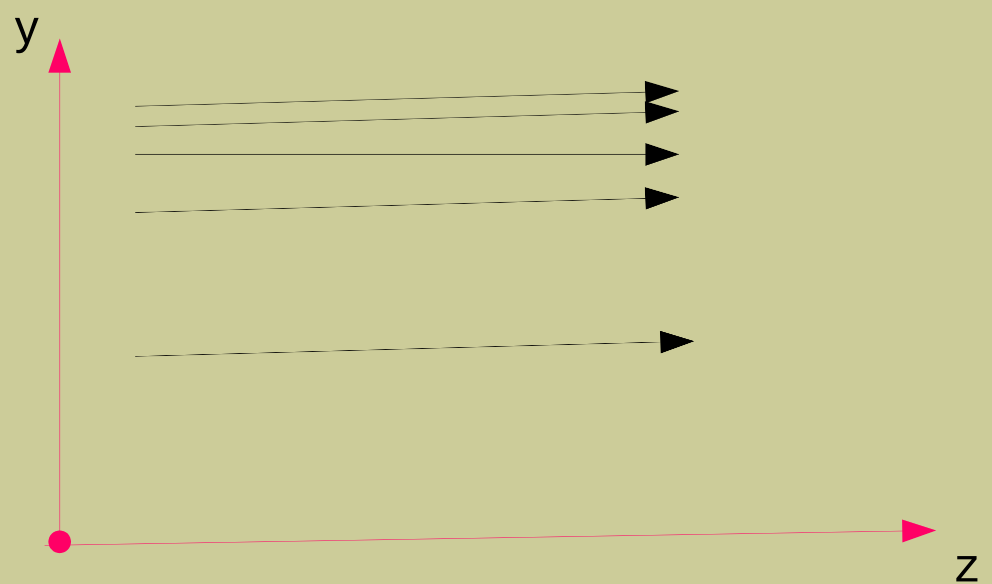

Figure shows the presence of gradient in magnetic field in y-direction.

$$F_x = q(v_y B_z)$$
$$F_y = -q(v_x B_z)$$
$$F_z = 0$$

$$B_z(y) = B_0 + y\frac{dB_z}{dy} + \dots$$

Expanding B around the origin which is assumed to be guiding center.

$$F_x = qv_y\left(B_0 + y\frac{dB_z}{dy}\right)$$

$$F_y = -qv_x\left(B_0 + y\frac{dB_z}{dy}\right)$$

$$F_x = -qv_\perp \sin(\omega_c t)\left(B_0 \pm r_L \cos(\omega_c t)\frac{dB_z}{dy}\right)$$

$$F_y = -qv_\perp \cos(\omega_c t)\left(B_0 \pm r_L \cos(\omega_c t)\frac{dB_z}{dy}\right)$$

We average force over a gyroperiod

$$<F_x> = 0.$$

$$\langle F_y \rangle = -qv_\perp\left[B_0\langle\cos(\omega_c t)\rangle \pm r_L\langle\cos^2(\omega_c t)\rangle\frac{dB_z}{dy}\right]$$

$$= \mp\frac{qv_\perp r_L}{2}\frac{dB_z}{dy}$$

Gradient Drift Motion

$$\mathbf{v}_{\nabla B} = \pm \frac{1}{2} v_{\perp} r_{L} \frac{\mathbf{B} \times \nabla B}{B^2}$$

Where ± stands for the sign of the charge.

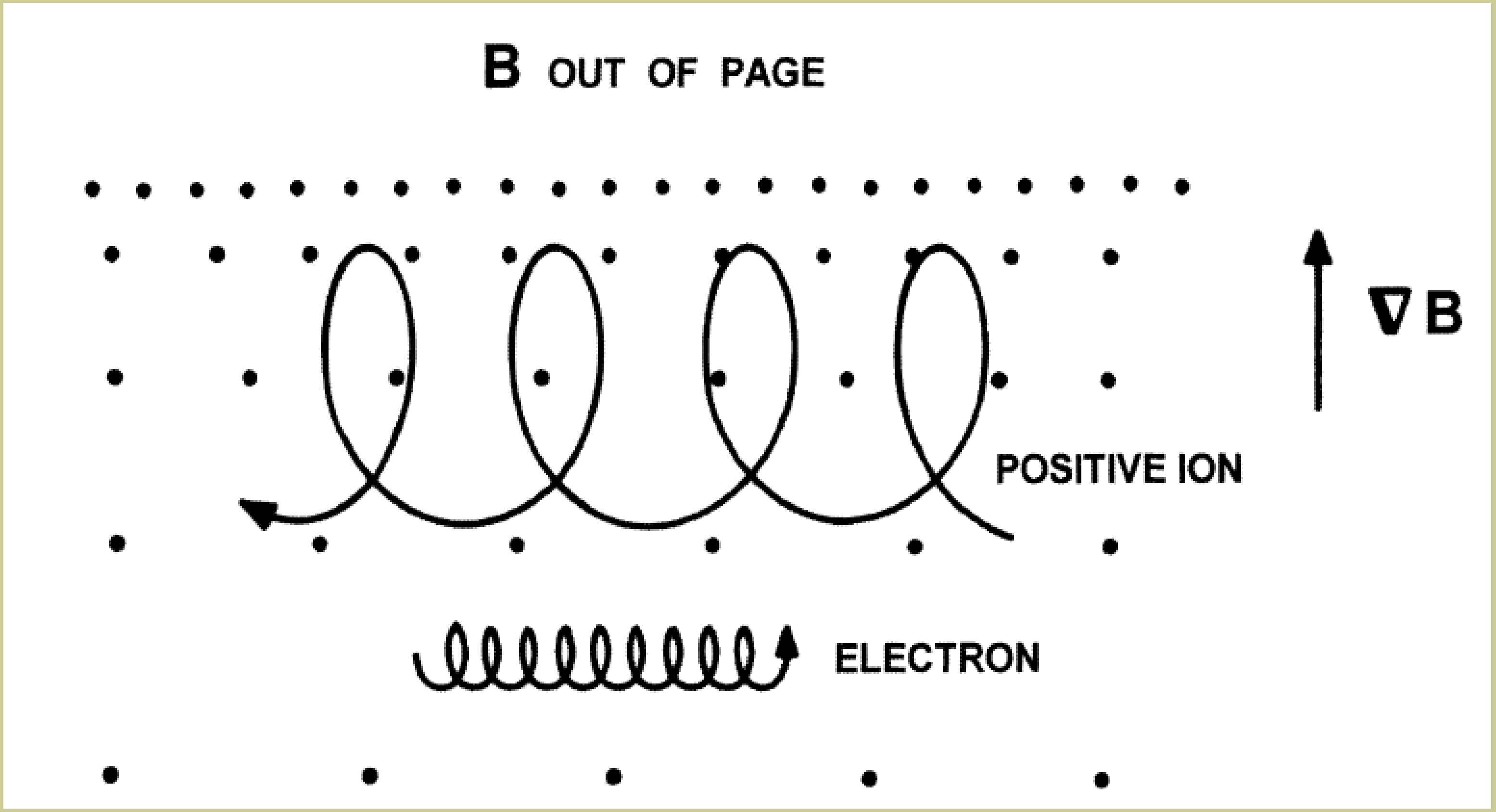

Figure is from “Fundament of Plasma Physics” by Bittencourt.

Thanks!