Charged Particle in Non-Uniform Magnetic Field

Course: MPHYEC-01I Plasma Physics (M.Sc. IV Sem)

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Lecture 5: Unit-I

Charged Particle in Non-Uniform Magnetic Field

- For uniform fields we were able to obtain exact expressions for the trajectory of the particle and the guiding center drifts.

- As soon as we introduce inhomogeneity, the problem becomes too complicated to solve exactly.

- Here we consider the motion of a charged particle in a static magnetic field which is slightly inhomogeneous in space i.e., we will focus on only those magnetostatic fields whose spatial change in a distance of the order of the Larmor radius, r_c, is much smaller than the magnitude of the field itself.

- The spatial change in the magnitude of magnetic field in a distance of the order of r_c

$$\delta B = r_c \, |\nabla B|$$

Then, the assuption is:

$$\delta B \ll B$$

- This approximation in often referred to as the **first-order orbit theory** or the **guiding center approximation.**

Lets consider a magnetic field $\mathbf{B}_0$ at origin and directed along the z direction

$$\mathbf{B}(0,0,0) \equiv \mathbf{B}_0 = B_0\hat{\mathbf{z}}$$

Let **r** be the momentary position vector of the particle. Then the magnetic field around **r** can be expressed by a Taylor expansion about the origin:

$$\mathbf{B}(\mathbf{r}) = \mathbf{B}_0 + \mathbf{r}\cdot(\nabla\mathbf{B}) + \quad \cdots$$

where the derivatives of **B** are to be calculated at the origin. The higher order terms can be neglected under the assumption:

$$\delta B = |\mathbf{r}\cdot(\nabla\mathbf{B})| \ll |\mathbf{B}_0|$$

$$\mathbf{r}\cdot(\nabla\mathbf{B}) = (\mathbf{r}\cdot\nabla)\mathbf{B} = \left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + z\frac{\partial}{\partial z}\right)\mathbf{B} =$$

$$\left(x\frac{\partial B_x}{\partial x} + y\frac{\partial B_x}{\partial y} + z\frac{\partial B_x}{\partial z}\right)\hat{\mathbf{x}} + \left(x\frac{\partial B_y}{\partial x} + y\frac{\partial B_y}{\partial y} + z\frac{\partial B_y}{\partial z}\right)\hat{\mathbf{y}} +$$

$$\left(x\frac{\partial B_z}{\partial x} + y\frac{\partial B_z}{\partial y} + z\frac{\partial B_z}{\partial z}\right)\hat{\mathbf{z}}$$

Note: Dependence of magnetic field components on x, y, z leads to non-zero values of their derivatives which cause the non-vanishing gradient, curvature, shear (twist) in the magnetic field (magnetic field lines).

Equation of motion:

$$m\frac{d\mathbf{v}}{dt} = q(\mathbf{v} \times \mathbf{B}_0) + q\mathbf{v} \times [\mathbf{r} \cdot (\nabla \mathbf{B})]$$

The particle velocity can be written as a superposition,

$$\mathbf{v} = \mathbf{v}^{(0)} + \mathbf{v}^{(1)} = \frac{d\mathbf{r}^{(0)}}{dt} + \frac{d\mathbf{r}^{(1)}}{dt}$$

where $\mathbf{v}^{(1)}$ is a first-order perturbation such that

$$|\mathbf{v}^{(1)}| \ll |\mathbf{v}^{(0)}|$$

and $\mathbf{v}^{(0)}$ is the solution of the zero-order equation:

$$m\frac{d\mathbf{v}^{(0)}}{dt} = q(\mathbf{v}^{(0)} \times \mathbf{B}_0)$$

Neglecting second-order terms we can write, therefore,

$$\mathbf{v} \times [\mathbf{r} \cdot (\nabla \mathbf{B})] = \mathbf{v}^{(0)} \times [\mathbf{r}^{(0)} \cdot (\nabla \mathbf{B})]$$

Under these approximations,

$$m\frac{d\mathbf{v}}{dt} = q(\mathbf{v} \times \mathbf{B}_0) + q\mathbf{v}^{(0)} \times [\mathbf{r}^{(0)} \cdot (\nabla \mathbf{B})]$$

Various kind of drift motions (like Gradient drift, Curvature drift) are produced.

Gradient Drift

$$\mathbf{v}_{\nabla B} = \pm \frac{1}{2} v_{\perp} r_{L} \frac{\mathbf{B} \times \nabla B}{B^2}$$

Where $\pm$ stands for the sign of the charge.

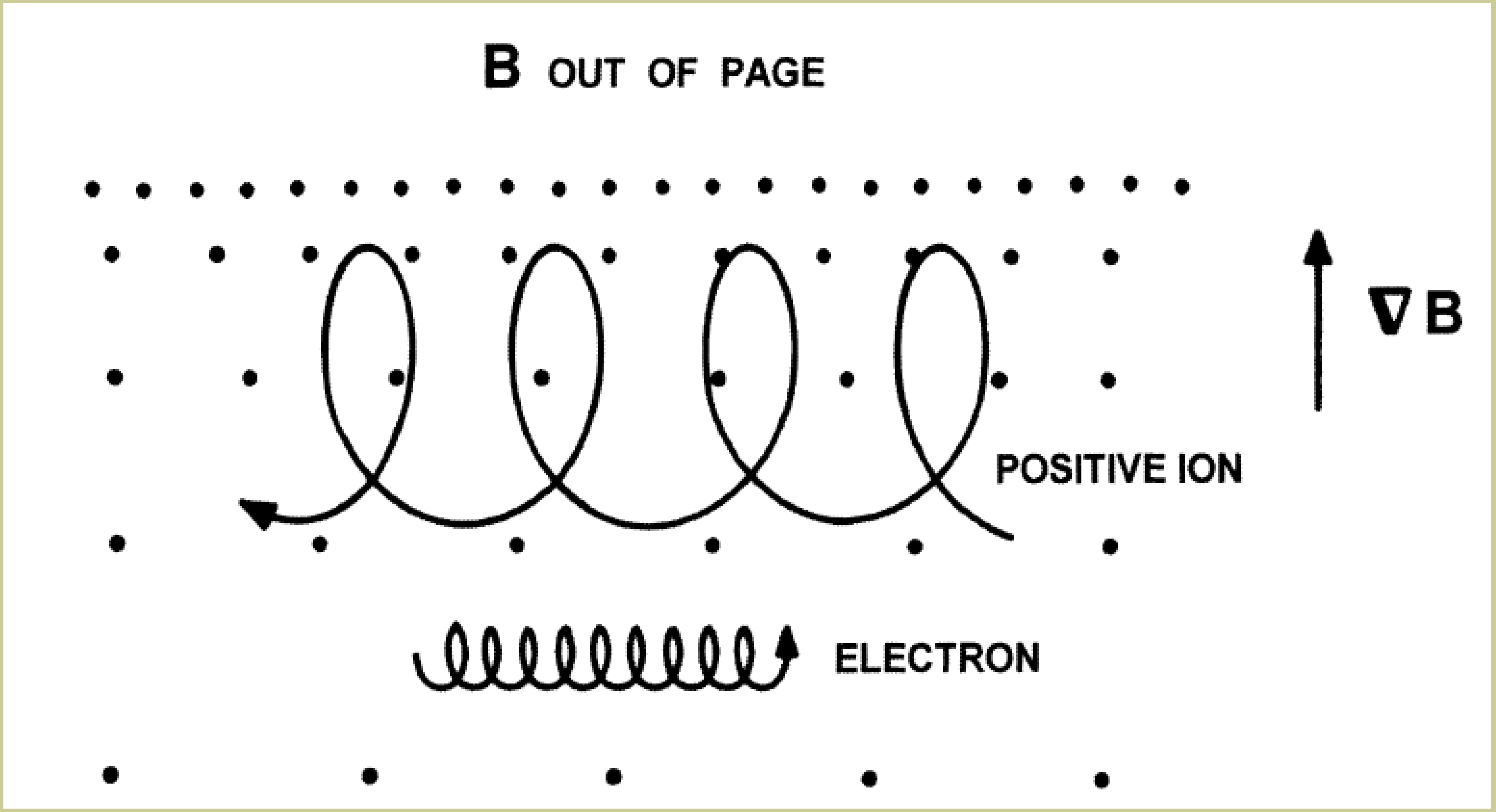


Figure is from “Fundament of Plasma Physics” by Bittencourt.

Comment on the drift motion due to an external force

Presence of some additional uniform force **F** modifies the equation of the charged particle (q) as:

$$m\frac{d\mathbf{v}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \mathbf{F}$$

If, for simplicity, we assume **E**=0

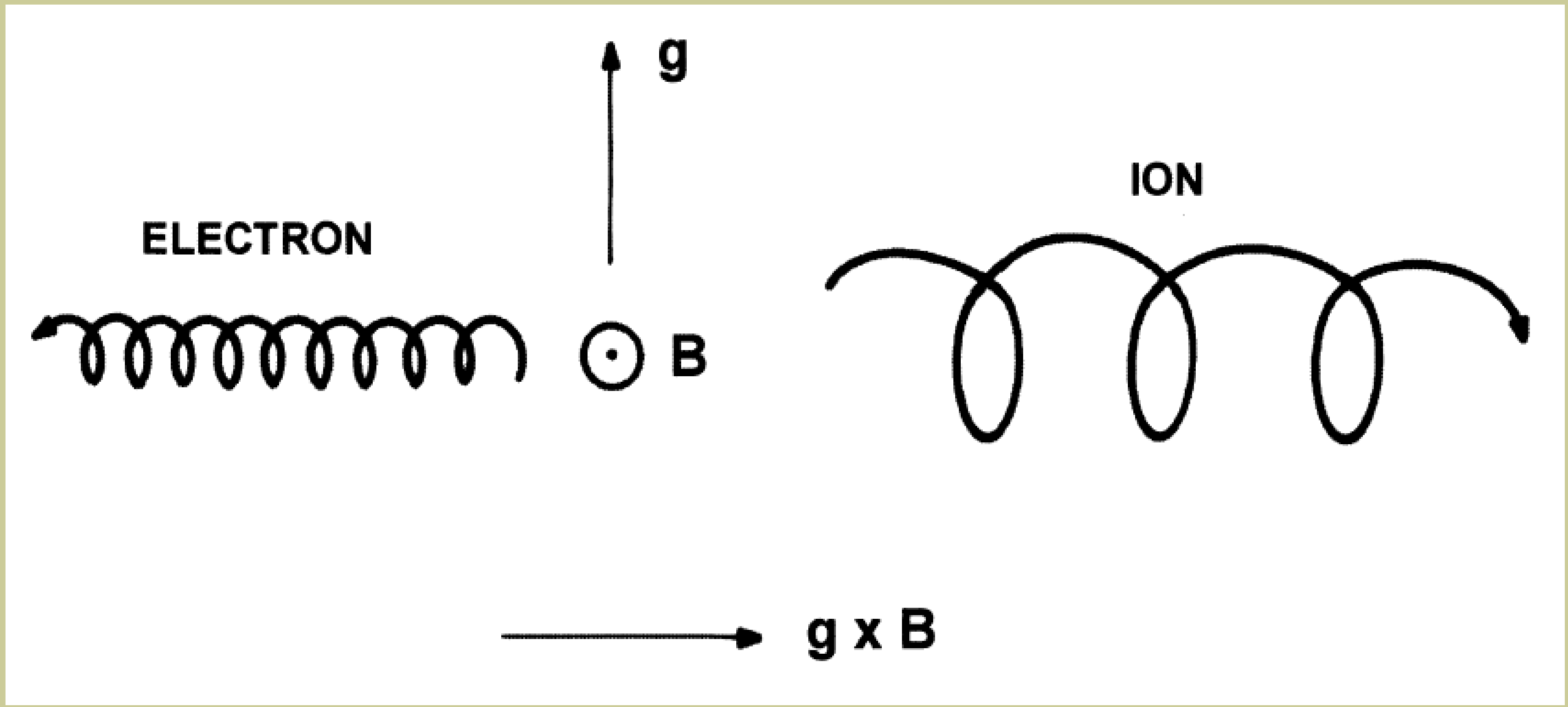
$$m\frac{d\mathbf{v}}{dt} = q(\cancel{\mathbf{E}} + \mathbf{v} \times \mathbf{B}) + \mathbf{F}$$

Now, in addition to magnetic force, we have force **F** which, like electric field, produces Drift motion of the guiding center. By anology with $\mathbf{v}_E$, we can write the drift velocity due to **F** ($\mathbf{v}_F$) as:

$$\mathbf{v}_F = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

Example, if we have the gravitational force, then **F**=m**g** and corresponding drift velocity is:

$$\mathbf{v}_g = \frac{m}{q}\frac{\mathbf{g} \times \mathbf{B}}{B^2}$$



The figure is from “Fundaments of Plasma Physics” by Bittencourt. Notably, Unlike **ExB** drift produced due to electric force, the drift motion due to gravitational force depends on the charge of the particle.

Thanks!